Information Disclosure and Partner Management in Affiliate Marketing

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ABSTRACT

The recent massive proliferation of affiliate marketing suggests a new e-commerce paradigm which involves sellers, affiliates and the platforms that connect them. In particular, the fact that prospective buyers may become acquainted with the promotion through more than one affiliate to whom they are connected calls for new mechanisms for compensating affiliates for their promotional efforts. In this paper, we study the problem of a platform that needs to decide on the commission to be awarded to affiliates for promoting a given product or service. Our equilibrium-based analysis, which applies to the case where affiliates are a priori homogeneous and self-interested, enables showing that a minor change in the way the platform discloses information to the affiliates results in a tremendous (positive) effect on the platform's expected profit. In particular, we show that with the revised mechanism the platform can overcome the multi-equilibria problem that arises in the traditional mechanism and can obtain a profit which is at least as high as the maximum profit in any of the equilibria that hold in the latter.

CCS CONCEPTS

• **Applied computing** → *Electronic commerce*; *Marketing*.

KEYWORDS

affiliate marketing, equilibrium, dynamic pricing, mechanism design

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1 INTRODUCTION

Affiliate marketing is a new e-commerce paradigm in which one can earn a commission by promoting other people's (or company's) products or services (either resulting from a click or from an actual sale) [17, 20]. The idea is that content producers can monetize their network by promoting products and services, hence saving manufacturers time and effort in reaching their target audience, and share some of the revenue. Many companies nowadays are offering promoters a financial incentive through an affiliate program. Affiliate marketing has become a very large industry and a key source of online income for many thousands of professional bloggers, celebrities and social media stars, ultimately creating passive income streams. According to a recent Business Insider report [23], by 2022, affiliate marketing content is projected to generate $8.2 billion in revenue in the US.

The process of affiliate marketing generally involves four parties. First there is the advertiser, who sells a product or service online, and the affiliate who generates content through which she can promote the product. The connection between the two is made through a platform (such as AWIN (www.awin.com), ShareASale (www.shareasale.com), Maxbounty (www.maxbounty.com), Tradedoubler (www.tradedoubler.com) and CJ Affiliate (www.cj.com)). The platform lists products and services requiring promotion and provides special links, known as affiliate links, which the affiliates use for directing prospective buyers to the product web page on the advertiser's website and for identifying the affiliate (usually implemented using cookies). The platform is also the one receiving the payment from the advertiser and gets to decide how much to offer the affiliates for their service [8]. Finally, there is the buyer who is usually unaware of these dynamics that take place between the advertiser, the platform and the affiliate.

Most research on affiliate marketing to date has focused on the benefits and the potential of this new mechanism [6, 8], pricing strategies for the advertiser [17], methods for recommending affiliates to advertisers [19, 21] and, fraud detection and prevention [2, 5, 24]. None of these works has considered affiliates' strategic considerations in their decision whether or not to promote a specific product.

In this paper we provide a game-theoretic based analysis of an affiliate marketing platform and affiliates. In particular, we focus on information disclosure, enabling disclosing the number of affiliates that have already become acquainted with a given opportunity. We show that even by revealing such minimal information, the platform can enforce an equilibrium of the highest possible expected profit among those that hold in the legacy mechanism used in practice in most platforms nowadays.

Contributions. We provide a comprehensive equilibrium analysis for an affiliate marketing model where affiliates are a priori homogeneous. To the best of our knowledge, this is the first attempt to date to provide a formal game-theoretic based analysis of such a model. Not only did prior analysis not take into consideration affiliates'

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strategic behavior (i.e., their choice of whether or not to promote a product), it also did not take into account the network effect in the sense that affiliates may have shared followers. Our analysis reveals that in many cases there is not one, but many possible equilibria. In such cases, the core equilibrium analysis does not provide a way to determine which of the equilibria will be adopted by the players. So, the platform cannot optimize the mechanism, and in particular has no way to determine the optimal payment to be offered to affiliates, since the effectiveness of such payments will depend on the exact equilibrium chosen by the players.

We show that a minor modification to the basic mechanism allows the platform to dictate the use of the equilibrium that yields the highest expected profit. This is obtained by using a dynamic commission structure and disclosing the number of affiliates that have already become acquainted with the opportunity to each affiliate considering it. Other than the important advantage of dictating the preferred equilibrium, the revised mechanism is also computationally highly efficient.

2 RELATED WORK

Despite its high applicability and presence in online markets, which is also reflected in business reports [23] and case studies [15], the number of studies directly touching on affiliate marketing is rather modest and it seems like research still has a long way to go in order to fully unveil the potential of this transformative mechanism. In particular there is a need for theoretical and empirical research covering all of the parties involved in affiliate marketing [9].

The focus of most existing research on affiliate marketing is the platform's or advertiser's choice of the payment scheme to be used (primarily the use of pay-per-lead, where the affiliate is paid for any potential customer directed to the advertiser's site, versus pay-per-conversion, where she is paid for the number of leads converted to customers) [11, 17], the advantages of switching to affiliate marketing [1] and the influence of different choices made in the mechanism's design (e.g., one-to-many and one-to-one with respect to the advertiser-affiliates relationship) over the expected profit [17].

While most of the existing work takes the advertiser's and the platform's perspective, few authors provide insights from the affiliate's point of view [4, 20]. This latter line of work, however, mostly relies on data analysis, surveys and interviews. For example, Benedictova and Nevosad [4] interview affiliate partners and propose subjective suggestions to the advertiser from the affiliate's point of view. Mizuno [20] simulates affiliate and buyer agents, basing the behaviors of the agents on surveys. Their simulation results attempt to suggest the right mixture of advertisement content in the affiliate's blog to increase her revenues. Distinct from the related work presented here, our paper frames and analyzes the strategic considerations of the affiliates, taking a game-theoretic approach.

In a way, affiliate marketing resembles referral programs [12, 22], where various pricing and referral strategies under different demand dynamics have been studied [13]. In a referral program, whenever a consumer makes a purchase, the firm gives her a link to share with friends, and every purchase coming through that link generates a referral payment. Still, the assumptions that prior work studying referral programs models have relied on, especially those related to the nature of the promotion made, are very different than those used for affiliate marketing. For example, Lobel et al [18] assume that in the referral program, the consumer needs to directly contact each friend, i.e., incurs a cost for every referral made. Therefore, the key decision for the consumer is how many friends to contact. In affiliate marketing, the affiliate incurs a one-time publication cost for exposing all of her followers to the opportunity. Furthermore, the analysis provided by Lobel et al is based on the assumption that the population of potential consumers is represented as a rooted graph, and that only the consumer that is the root of this graph is approached by the firm. From that point on, the process follows the dynamics of a pyramid. In affiliate marketing, on the other hand, affiliates join only through accessing the platform, and there is no a priori advantage to any affiliate in the sense of getting the information before the others.

Other than the above reviewed work, in recent years much focus has been put on studying tangential web-based marketing models which are somehow related to our model. These include cashback-based affiliate marketing (i.e., getting a cashback whenever making the purchase through a specific affiliated (rather than through the original) site [3, 14]), multi-level marketing (MLM) through social influence networks [10] and the management of early adopters, taking into account their social influence, when launching a new product [7, 16].

3 MODEL

Our model considers an affiliate marketing platform and N prospective affiliate partners (denoted "partners" henceforth). The platform offers a product or a service requiring promotion and provides its affiliate link, which the partners can use for directing their followers (connections/friends who are the potential buyers) from their social network to the product's web page on the advertiser's website. The model assumes that the partners are a priori homogeneous, in the sense that each of them is connected to k followers and each of the latter is potentially connected to $1 \leq w \leq N$ partners overall, where w is a priori unknown and characterized by a probability function $p_W(w)$ such that $\sum_{w=1}^{N} p_W(w) = 1$ (see Figure 1).[1]

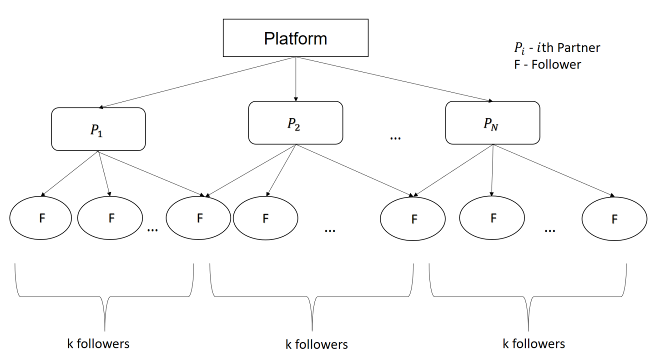

Figure 1: Network Structure - Each partner has k followers with some overlap.

[1]Alternatively, we can assume that the number of potential buyers of each partner is probabilistic. This would require minor changes in the analysis and will slightly change the claims and proofs. However, this will require substantially more technical work in the equations.

A partner can either promote the product or opt not to promote it (see Figure 3 in Appendix A for an illustration). Promoting incurs a cost c, whereas not promoting does not incur any cost.[2] The model assumes that upon promoting, the content will reach all of the partner's followers, and even if a follower does not read it right away she will review the promotions in the order received (e.g., in case she is connected to several partners that promote the product). In order to encourage a partner to promote the product, the platform offers a commission (in the form of a fixed payment) M for each lead that will convert to a purchase. We assume that a random follower will be interested in the promoted product (to the level of purchasing it) with probability p_B[3]. A follower will not use the affiliate link in case another link to the same product has already been received from one of the other partners to whom she is connected. Finally, the platform's gain from each successful sale of the product is denoted G.

All of the players (platform and partners) are assumed to be fully rational and self-interested in the sense that their goal is to maximize their individual expected profit. The expected profit of the platform is its gain from successful sales minus the commissions paid to partners. The expected profit of a partner is zero if not promoting the product and otherwise it is the expected total commissions received minus the cost of promotion. Finally, the model assumes that all of the players are familiar with the parameters M, p_B, N, k, c and the function $p_W(w)$. This is a relatively genuine reflection of real-life settings, as most of this information is either publicly available or can be found with minimal effort.[4]

4 ANALYSIS

We begin with analyzing the affiliate marketing mechanism in its core form, i.e., without disclosing any information other than the listing itself, as applied by platforms nowadays. Then we analyze the slightly modified mechanism in which partners-views information is being disclosed to partners, and its dynamic pricing extension. Synthetic numerical examples are used, whenever applicable, for illustration purposes.

4.1 Core Mechanism

When the platform simply lists the details of the product and generates affiliate links to partners, the setting can be considered a simultaneous game. Meaning that even though the partners do not necessarily all access the listing at the same time, they are unaware of how many of the other partners will decide to promote it.

A partner's strategy can thus be captured by the probability $0 \leq p \leq 1$ she will choose to promote the product. A pure-strategy Nash equilibrium is thus one where each partner P_i is using $p_i \in \{0, 1\}$. A mixed-strategy Nash equilibrium is the one where at least one partner P_i is using $0 < p_i < 1$. One natural and highly intuitive equilibrium that always holds is the symmetric equilibrium when all partners are using the same strategy p.[5] For exposition purposes, we use this equilibrium whenever considering a mixed-strategy equilibrium, alas the proofs we provide regarding the dominance of the equilibrium of the proposed modified mechanism over mixed-strategy equilibria of the core mechanism hold also for all types of mixed-strategy equilibria.

Consider a solution according to which i partners out of N choose to promote. The expected number of followers exposed to the product, denoted $Expose(i)$, is given by

$$Expose(i) = ik\sum_{w=1}^{N} p_W(w) \sum_{z=max(0,w-N+i-1)}^{min(w-1,i-1)} \frac{\binom{i-1}{z}\binom{N-i}{w-1-z}}{\binom{N-1}{w-1}} \frac{1}{z+1}$$

The calculation iterates over the k followers of each of the i partners. For each follower it considers all of the possible partners to whom she is connected, w. Then it calculates the probability of having z of the other $i-1$ promoting partners be part of the set of $w-1$ other partners to whom the current follower is connected. To eliminate this redundancy in counting, we divide by $z+1$.

The expected profit of a promoting partner if the total number of promoting partners (including herself) is i, denoted $B^P_{core}(i)$ is thus:

$$B^P_{core}(i) = -c + \frac{Expose(i)Mp_B}{i} \tag{1}$$

Hence the best response strategy of a partner is to promote if $B^P_{core}(i) \geq 0$ (as not promoting results in zero profit).

Similarly, the expected profit of the platform when the total number of promoting partners is i, denoted $B^{platform}_{core}(i)$, is:

$$B^{platform}_{core}(i) = Expose(i)(G - M)p_B \tag{2}$$

The analysis of the symmetric mixed strategies is quite similar. The expected profit of the platform when the partners use a mixed strategy p, denoted $B^{platform}_{core}(p)$, is:

$$B^{platform}_{core}(p) = \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} Expose(i) \cdot (G - M)p_B \tag{3}$$

Similarly, the expected profit of a promoting partner, if all other partners are promoting with probability p, denoted $B^P_{core}(p)$, is:

$$B^P_{core}(p) = -c + \sum_{i=0}^{N-1} \binom{N-1}{i} p^i (1-p)^{N-i-1} \frac{Expose(i+1)Mp_B}{i+1}$$

However, a simpler expression can be obtained by considering the number of other partners each of the followers has (see Appendix B for the detailed mathematical manipulations used):

$$B^P_{core}(p) = -c + k\sum_{w=1}^{N} p_W(w) \cdot \sum_{z=0}^{w-1} \binom{w-1}{z} \frac{p^z(1-p)^{w-z-1}}{z+1} Mp_B \tag{4}$$

$$= -c + k\sum_{w=1}^{N} p_W(w) \frac{(1-(1-p)^w)}{wp} Mp_B$$

The above enables characterization of the equilibrium. The best response strategy of every partner is: (a) to promote if $B^P_{core}(i+1) \geq$

[2] All partners are characterized with the same promotion cost as this is usually the (quite standard) cost of time it takes for uploading a post or the reputation loss associated with promoting the product in their content.

[3] Notice that by setting $p_B = 1$ the model changes into (and the analysis and all proofs become applicable to) a pay-per-lead scheme.

[4] For example, it is easy to know how many readers a blog post has reached [20] or to predict exposure of future posts.

[5] For example, Lobel et al use this kind of equilibrium in their referral programs based model [18].

0 whenever the other partners are using a pure strategy according to which i of them promote; and (b) to promote if $B^P_{core}(p) \geq 0$ whenever the other partners are promoting with probability $0 < p < 1$. A pure-strategy Bayesian Nash Equilibrium (BNE) solution where $i \leq N$ promote is thus one where $B^P_{core}(i) \geq 0 \geq B^P_{core}(i+1)$, i.e., neither promoting nor non-promoting partners have an incentive to deviate. A symmetric BNE solution p is one where $B^P_{core}(p) = 0$. The above analysis can be augmented to accommodate solutions where some of the partners use pure strategies and some use mixed ones. The extension is quite mathematically technical and does not contribute much in terms of results, therefore it is omitted.

Proposition 4.1. *Any i-based promoting partners pure-strategy equilibrium of the core mechanism in which the partners are making zero profit (i.e., $B^P_{core}(i) = 0$ according to (1)) results in at least as high expected profit (to the platform) as any other i-based pure-strategy equilibrium in which the partners' expected profit is positive.*[6]

Proof. This derives from the fact that the platform's gains from the partners' promotions are the same (and equal $Expose(i) \cdot p_B \cdot G$), whereas in the case where partners' profit is zero, the expected sum of commissions paid is necessarily lower than when they make a profit (as the partners' gain derives solely from the platform's commission). □

Based on Proposition 4.1, we can calculate the expected-profit-maximizing M value by setting $B^P_{core}(i) = 0$ and solving (1) for any i (and calculating the corresponding expected profit $B^{platform}_{core}(i)$ according to (2)).

Figure 2 depicts the platform's profit with the mixed and pure equilibria as a function of the commission used, M, for a setting with $N = 50$ partners. The promotion cost is $c = 20$, the gain from a purchase is $G = 15$, the probability of a purchase is $p_B = 0.1$, the number of followers of each partner is $k = 25$ and the probability function of the number of partners to whom a follower is connected is given by $p_W(w) = 1/3$ for $1 \leq w \leq 3$, and $p_W(w) = 0$ otherwise. While the mixed-strategy equilibrium is continuous (in M), the pure-strategy equilibrium exhibits a recurring pattern of a sudden increase (a step-function) followed by a continuous decrease. The increase is associated with a transition from an equilibrium based on i promoting partners to one with $i + 1$. The decrease is when the number of promoting partners remains the same, yet the increase in M reduces the profit gained from any purchasing follower.

Interestingly, as observed from the figure, neither of the equilibria generally dominate the other in terms of the platform's expected profit for any given M. Later on, we prove that if M is within the control of the platform, as is often the case in most real-world settings, the pure-strategy equilibrium yields the maximum expected profit for the platform.[7]

4.2 Proposed Modified Mechanism

While with the core mechanism the platform cannot influence the equilibrium that will hold in a multi-equilibria scenario, we propose a simple modification to the mechanism. With the modified mechanism only one equilibrium holds. The idea is that the platform will provide every partner that accesses the listing with the number of other partners that have already accessed it.[8] This small change turns the game into a sequential one, as it enables each partner some additional information. Therefore, while in the core mechanism a pure strategy of a partner is simply a binary decision - whether or not to promote, here the strategy of partner P_i is a *function*, determining whether or not to promote given the information about the number of other partners that have accessed the listing so far.

[6]An i-based promoting partners pure equilibrium necessarily exists for any $i > 0$, as the increase in $Expose(i)$ due to an increase in i is a decreasing function.

[7]An example where M is not fully within the control of the platform is when the platform offers a fixed M for all products or services listed on its website.

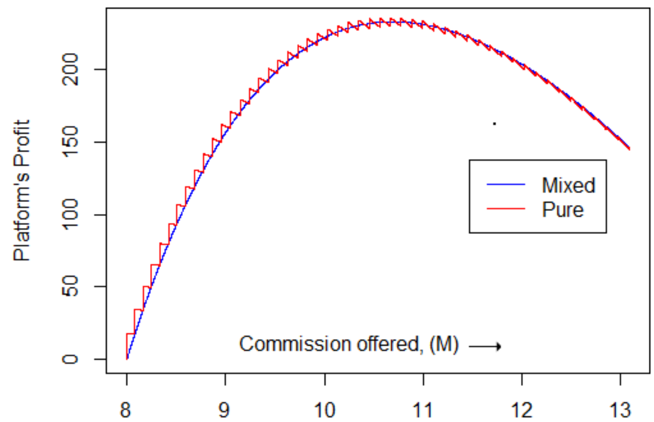

Figure 2: The platform's expected profit with the pure-strategy and symmetric mixed-strategy equilibria for different values of the commission offered (M).

The modified mechanism (denoted "sequential" henceforth) is a dynamic game. As such, the proper, or natural solution concept for this model is the sub-game perfect Nash equilibrium (SPNE). We note that since the sequential game is a game of complete information, the SPNE can be readily computed using backward induction. Note that except for the relatively rare case of ties, the sub-game perfect Nash equilibrium is unique and uses pure strategies.

We use $Expose^{marginal}(i)$ to denote the expected number of new followers becoming exposed to the product as a result of the ith promotion. Formally,

$$Expose^{marginal}(i) = Expose(i) - Expose(i-1)$$

The expected profit of a partner who is the ith promoting partner is therefore:

$$B^P_{seq}(i) = -c + Expose^{marginal}(i) M p_B \tag{5}$$

Proposition 4.2. *The SPNE for the sequential mechanism is to have each partner participate if the number of other partners who accessed the listing is $n' < n^*$, where $n^* = \lfloor n \rfloor$ such that n is the solution to $B^P_{seq}(n) = 0$ (according to (5)), and avoid participation otherwise.*

Proof. Consider any partner who receives information $n' < n^*$. In this case, the expected profit if promoting is at least $B^P_{seq}(n'+1)$,

[8]Meaning that we do not even need to provide information about how many others have received an affiliate link. Instead we only provide information about how many times the listing was uniquely viewed.

as $B^P_{seq}(i)$ decreases in i and the number of promoting partners so far is at most n'. Also: $B^P_{seq}(n'+1) \geq B^P_{seq}(n^*) \geq 0$. Therefore promoting is the dominating strategy. Therefore all partners receiving information $n' < n^*$ will promote. Now consider a partner receiving information $n' \geq n^*$. Knowing that the first n^* partners who viewed the listing necessarily promoted it, the partner will find not promoting to be the dominating strategy. □

The expected profit of the platform can be calculated using (2), substituting $i = n^*$.

Unfortunately, the sequential mechanism as presented above falls behind pure-strategy Nash equilibria of the core mechanism, as stated in Proposition 4.3.

Proposition 4.3. *For any M, the SPNE of the sequential mechanism is weakly dominated by at least one pure Nash equilibrium of the core mechanism that uses the same M.*

Proof. In the sequential case in equilibrium there are exactly n^* partners choosing to promote. We show that any pure equilibrium that holds with the core mechanism is based on at least n^* promoting partners.

Consider the n^*th promoting partner in the sequential mechanism. The expected profit of this partner $B^P_{seq}(n^*)$ is non-negative, based on Proposition 4.2. Furthermore, $B^P_{seq}(n^*) = -c + Expose^{marginal}(n^*)Mp_B = -c + (Expose(n^*) - Expose(n^*-1))Mp_B < -c + Expose(n^*)Mp_B/n^* = B^P_{core}(n^*)$, as $Expose(i)$ increases at a decreasing rate in i. Therefore $B^P_{core}(n^*) > 0$, meaning that there are at least n^* partners promoting in any pure-strategy equilibrium in the core model. Therefore, since the platform's expected profit increases in the number of followers being exposed to the product (as both in the core and sequential mechanisms the same M is used), the pure-strategy equilibrium with the core mechanism offers at least the same expected profit as the SPNE of the sequential mechanism. □

4.3 Using Dynamic Commission

Proposition 4.3 suggests that the sequential mechanism offers no advantage as far as the platform's expected profit is concerned if the commission offered is fixed. Fortunately, the sequential mechanism can be further revised in a way that its SPNE will yield the same expected profit as the equilibrium associated with the maximum expected profit in the core mechanism. This is achieved by replacing the fixed commission M with a changing commission ("dynamic commission"), such that the ith approaching partner will receive a commission $M_i \ \forall i \leq N$. By properly setting the dynamic commission, the platform can take over the entire partners' surplus.

Theorem 4.4. *The sequential mechanism with dynamic commission will result in an SPNE with the maximum expected profit to the platform when setting the commission for the ith querying partner to $M_i = min(\frac{c}{Expose^{marginal}(i)p_B}, G)$.*[9]

[9] To be completely accurate, any other commission function according to which a subset of j arriving partners, where j is the integer part of the solution j' to $\frac{c}{Expose^{marginal}(j')p_B} = G$, are being offered $\frac{c}{Expose^{marginal}(l)p_B}$ (where l is their order of arrival within the sequence) and the remaining partners being offered zero will result in the same expected profit in its SPNE.

Furthermore, the platform's expected profit with this SPNE will be at least as high as with any pure-strategy Nash equilibrium in the core mechanism. In particular, when the platform has full control over M in the core mechanism, the expected profit with the SPNE of the sequential mechanism will be equal to the expected profit obtained with the Nash equilibrium yielding the maximum expected profit in the core mechanism.

Proof. First we prove that offering a commission $min(\frac{c}{Expose^{marginal}(i)p_B}, G)$ to the ith querying partner is optimal in the sequential mechanism. Assume otherwise, i.e., the platform uses a different commission structure. Obviously, the commission to the ith querying partner affects only the decision of that partner. Now consider a partner P_i being offered a commission $M'_i < min(\frac{c}{Expose^{marginal}(i)p_B}, G)$. Here the partner will opt not to promote, as otherwise her expected profit is negative, resulting in zero contribution to the platform's profit. With a commission $min(\frac{c}{Expose^{marginal}(i)p_B}, G)$ the partner will either promote (in case $\frac{c}{Expose^{marginal}(i)p_B} < G$) or not promote (otherwise). Since the commission is bounded by G, the contribution to the platform's profit when choosing to promote is necessarily non-negative. Similarly, consider a partner P_i being offered a commission $M'_i > min(\frac{c}{Expose^{marginal}(i)p_B}, G)$. Here, by reducing the commission to $min(\frac{c}{Expose^{marginal}(i)p_B}, G)$, the partner's decision whether or not to promote will remain the same, yet in case of promoting, the payments to be made to the partner will be smaller.

Next, we show that for any pure-strategy equilibrium with i promoting partners (that make zero profit) in the core mechanism, the platform can achieve the exact same expected profit by using commissions $\frac{c}{Expose^{marginal}(i)p_B}$ for the first i participating partners and G for the remaining ones. The expected profit from those receiving a commission $\frac{c}{Expose^{marginal}(i)p_B}$ is the same as with the core mechanism as in both cases we obtain i promoting partners and the commission they receive equals their costs. The expected profit from those offered a commission G is zero. The expected profit with the sequential mechanism can be further improved by switching to the commission structure dictated by Theorem 4.4, as we are either giving up on partners yielding a negative expected marginal profit (when $\frac{c}{Expose^{marginal}(i)p_B} > G$) or adding partners yielding a positive profit (when $\frac{c}{Expose^{marginal}(i)p_B} < G$). This, together with Proposition 4.1, suggests that the sequential mechanism will result with at least the same expected profit as the core mechanism.

Finally, we show that when the expected-profit-maximizing M value is used in the core mechanism, the resulting pure-strategy equilibrium yields the same expected profit as the profit obtained with the sequential model. This derives from the fact that in the core mechanism for any $i \leq N$ there is a pure-strategy equilibrium in which there are i partners promoting (and the others do not) where the expected profit of all partners is zero. Therefore, if picking the M value that results in i-promoting-partners equilibrium such that i is the highest number of partners for which $\frac{c}{Expose^{marginal}(i)p_B} < G$ in the sequential mechanism, we get the same number of promoting partners with both mechanisms and the expected sum of commissions paid in both equals $c \cdot i$. Therefore the platform's expected profit in both is equal. □

The expected profit of the platform is therefore (setting the commission using Theorem 4.4):

$$B_{seq}^{platform} = \sum_{i=1}^{N} Expose^{marginal}(i)(G - M_i)p_B \tag{6}$$

While Theorem 4.4 proves that the sequential mechanism guarantees the maximum expected profit that can be obtained with pure-strategy equilibria in the core mechanism, the choice of using the former will depend on whether or not it also dominates mixed-strategy equilibria of the latter. Theorem 4.5 proves that indeed such domination holds.

Theorem 4.5. *Any mixed strategy equilibrium of the core mechanism is strictly dominated (as far as the platform's expected profit is concerned) by the SPNE of the sequential mechanism with dynamic commission.*

Proof. A mixed equilibrium results in an induced distribution over the number of promoting partners, as captured in (3). The expected overall cost incurred by the partners when $i \leq N$ of them choose to promote is $c \cdot i$ and consequently the expected overall cost given the induced distribution is $\sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} c \cdot i$. Recall that the expected profit of any partner P_i which mixes between promoting and not promoting is zero. Meaning that the expected commission payment made by the platform, according to (3), is equal to the overall expected cost for the partners, formally: $\sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} c \cdot i = \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} Expose(i) \cdot Mp_B$. Therefore the platforms' expected profit can be expressed as: $B_{core}^{platform}(p) = \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} (Expose(i)Gp_B - c \cdot i)$.

Now consider the expected profit with the sequential mechanism with dynamic commission. Here as well the platform fully covers the promoting partners' expected cost, which equals $c \cdot i$, and its expected gain is $Expose(i)Gp_B$. However, unlike with the mixed equilibrium case, here the platform gets to explicitly choose the number of partners i that will promote, through the structure of the commission it offers. Formally, it chooses an i that maximizes $Expose(i)Gp_B - c \cdot i$. Finally, we obtain that $max_i(Expose(i)Gp_B - c \cdot i) > \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} (Expose(i)Gp_B - c \cdot i)$, i.e., the expected profit with the sequential mechanism is greater than with the mixed strategy equilibrium of the core mechanism. □

Notice that the above proof also holds for the case where a non-symmetric mixed-strategy equilibrium is used in the core mechanism, and even for the case where some of the partners use pure strategy and some mix. In both cases we still get a distribution over the number of participating partners. The only difference is that those partners using a pure strategy in the core mechanism are actually making a positive profit (i.e., requiring a greater payment from the platform's side). Everything else in the proof remains the same.

Theorem 4.5 has two important implications. First, since according to Theorem 4.4 the SPNE of the sequential mechanism provides the same expected profit as the best pure-strategy Nash equilibrium of the core mechanism, and as the former dominates any mixed-strategy equilibrium that holds in the core mechanism, then when having full control over the commission offered M, the platform will always prefer a pure-strategy Nash equilibrium over any mixed-strategy equilibrium. Second, the platform can guarantee that profit simply by switching to the sequential mechanism with the dynamic commissions.

We note that even in cases where the platform is forced to use the core mechanism, the sequential mechanism can be used to facilitate the calculation of the M value that will maximize the expected profit of the platform if the pure-strategy equilibrium is to be used. Simply solve (1) taking i to be the integer part of the solution to $\frac{c}{Expose^{marginal}(i)p_B} = G$. The latter i value is the number of promoting partners according to the sequential mechanism with dynamic commission. Therefore solving (1) with that i will guarantee the use of the pure-strategy equilibrium that results in the same expected profit as the one achieved with the latter mechanism, and according to Theorem 4.4 it is the expected-profit maximizing equilibrium.

5 DISCUSSION AND CONCLUSIONS

The equilibrium analysis provided in the paper is the first, to the best of our knowledge, to consider the strategic choices of partners in affiliate marketing while taking into account the complete set of influencing factors that hold in real-life. These include the information disclosed by the platform, the promotion costs and the possible overlap between the followers, as is the case in social networks. In fact, this is the first attempt to study information design in the context of affiliate marketing. Indeed, much like in many other game-theoretic analyses of markets, the model relies on the assumption that partners are a priori homogeneous. Still, as argued throughout the paper, the extension to the heterogeneous case is mostly technical and most of the results provided in the analysis section will hold, qualitatively.

The proposed mechanism, according to which information about the number of times an opportunity has been reviewed is disclosed to the partners and the commission offered is dynamic, is both easy to implement and encapsulates several important inherent advantages. The primary advantage is of course the strong guarantee to obtain an expected profit that equals the one obtained with the most profitable equilibrium among those that hold in the core mechanism in a multi-equilibria scenario. Others relate to computational aspects and the nature of the equilibrium (SPNE vs. NE). Furthermore, the sequential mechanism does not require the platform to determine an order in which partners review opportunities. Instead, they are serviced based on the order of arrival, hence no fairness issues arise.

We note that an alternative way for enforcing the expected-profit maximizing pure-strategy equilibrium in the core mechanism is to simply limit the number of partners who can promote (and set the commission accordingly, such that all promoting partners end up with no profit), as discussed in the former section. Yet, limiting the number of partners that can promote a given product is probably less appealing for the platform compared to simply providing information on how many have already reviewed the opportunity, as it may make partners reluctant to subscribe to the platform (which is bad, especially when some subscription fee is charged, as in some of the platforms).

We see many directions for extending this work. One natural direction is the analysis of multi-platform competition. This may become especially important as many platforms nowadays are becoming exclusive (due to contracts with manufacturers). Another interesting direction is the study of how network structure (number of partners, number of followers and distribution of shared followers) influences performance with the two mechanisms.

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A PLAYERS' CHOICES

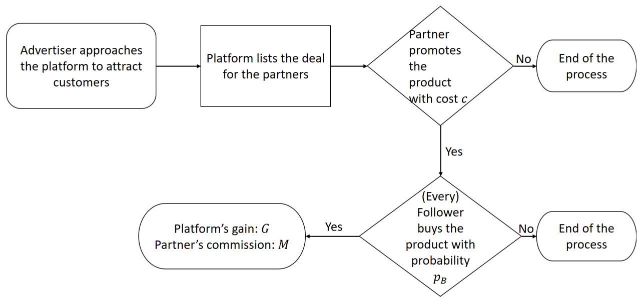

Figure 3: The choices of partners and followers in the model, and the resulting payments and costs incurred.

B CALCULATING $B^P_{core}(p)$

An alternative calculation of $B^P_{core}(p)$, the expected profit of a promoting partner if all other partners are promoting with probability p, is the one which considers the number of other partners to whom each of her followers is connected:

$$B^P_{core}(p) = -c + k\sum_{w=1}^{N} p_W(w)\cdot\sum_{z=0}^{w-1}\binom{w-1}{z}\frac{p^z(1-p)^{w-z-1}}{z+1}Mp_B$$
$$= -c + k\sum_{w=1}^{N} p_W(w)\frac{1}{w}\cdot\sum_{z=0}^{w-1}\binom{w}{z+1}p^z(1-p)^{w-z-1}Mp_B$$
$$= -c + k\sum_{w=1}^{N} p_W(w)\frac{1}{w}\cdot\sum_{z=1}^{w}\binom{w}{z}p^{z-1}(1-p)^{w-z}Mp_B$$
$$= -c + k\sum_{w=1}^{N} p_W(w)\frac{1}{wp}\cdot\sum_{z=1}^{w}\binom{w}{z}p^{z}(1-p)^{w-z}Mp_B$$
$$= -c + k\sum_{w=1}^{N} p_W(w)\frac{(1-(1-p)^w)}{wp}Mp_B$$

Here we iterate over the k followers of the partner, considering for each of them the number of partners w to whom she is connected. For each w we calculate the probability that z of the $w-1$ other partners are promoting (given that each one promotes with probability p), in which case the partner has a chance of $1/(z+1)$ to be the one benefiting from the promotion to that follower.